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Spiral trajectory in the horizontal Brazil nut effect

Fei Fang Chung, Chia-Yi Ju, and Sy-Sang Liaw
Department of Physics, National Chung-Hsing University, 250 Guo-Kuang Road, Taichung, Taiwan
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An intruder to a group of identical beads contained in a circular plate which is subjected to a circular vibration will trace approximately a cyclic spiral. The trajectory is a result of both migration and rotation. The intruder migrates in the radial direction while rotating with a constant speed with respect to the center of mass of the whole group of beads. The rotation velocity is due to friction between the beads and the container wall and determined by the vibration amplitude and the number of beads. The migration direction is dependent on the size ratio and mass ratio of the intruder to the background beads. The migration speed is constant for the outward migration, but decreases gradually when the intruder migrates toward the center of mass of the whole cluster for the inward case.

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I. INTRODUCTION

Granular materials are fascinating because they have obvious solidlike properties but behave like a fluid under certain circumstances. For the same reason, their behavior is difficult to study because neither Newtonian mechanics nor fluid dynamics can be employed directly. In many cases, the number of granular particles is so large that a collective behavior emerges, but still too small for this behavior to be described by mean field theory. The Brazil nut effect (BNE) is one such system that has been studied much experimentally [1–5] and theoretically [6–16]. In BNE, a large intruder rises all the way to the top of a group of small particles under vertical vibration [6]. It was also reported that a large intruder can dive to the bottom if it has a large enough mass [1,10]. In 2005, Schnautz *et al.* found that beads in a circular plate under horizontal swirling motion behave similarly [17]. Namely, a large bead will migrate either to the center or to the border of the plate. In this horizontal version of the BNE system, another interesting phenomenon was previously reported [18], in which the spin angular velocity of the cluster of beads decreases with increasing packing density. Even more amazingly, at a certain critical packing density, the angular velocity becomes negative, i.e., the cluster and the plate rotate in opposite directions. In this paper we study horizontal BNE using molecular dynamics simulation. We explain how a collective motion emerges and determine the angular velocity of the cluster and the migration velocity of the intruder.

A typical setup of circular horizontal BNE [17] is shown in Fig. 1. 375 small plastic beads of radius $d=0.3$ cm and mass $m=0.39$ g are placed in a circular plate of radius $R=6.9$ cm. The plate is driven by a swirling motion—that is, the center of the plate is rotating horizontally with respect to a fixed point—of frequency $w_p/2\pi=1.2$ Hz and amplitude $A=1.8$ cm. A steel intruder of radius $D=0.67$ cm and mass $M=10.04$ g was initially placed near the border of the plate. It was observed to migrate toward the center of the plate. The motion of the beads was recorded by a camera so that their trajectories can be traced as shown in Fig. 1 for the intruder and one of the plastic beads.

We simulated the motion of the beads by treating collisions as a linear spring force [19,20]. The inelastic collisions between beads are characterized by the normal restitution coefficient $\varepsilon<1$ in the simulation. The normal restitution coefficient between the beads and the border is assumed to have the same ε for simplicity. Beads in the plate are subjected to tangential frictions from the plate bottom and border. In our simulation, we found that the friction with the plate bottom is not essential in the sense that for any value of this friction from 0 to a certain value, the general behavior of the beads are qualitatively the same as what we observed in experiments. We thus simply neglect this friction in our simulations. Friction with the plate border, on the other hand, has to be considered appropriately so that the trajectories of beads look similar to what we observed in the laboratory. We assumed the friction between a bead and the plate border is either a value proportional to v_t, the relative tangential velocity between them, or a Coulomb type proportional to the spring force F, whichever is smaller [18]. Specifically, we took the friction to be $f=\min(\mu_k m|v_t|,\mu_C|F|)$ with $\mu_k=10{,}000\ \mathrm{s}^{-1}$ and $\mu_C=0.5$ [18].

II. COLLECTIVE MOTION

With appropriate amplitude A and number of beads N in the plate, we found that the trajectory of any plastic bead is

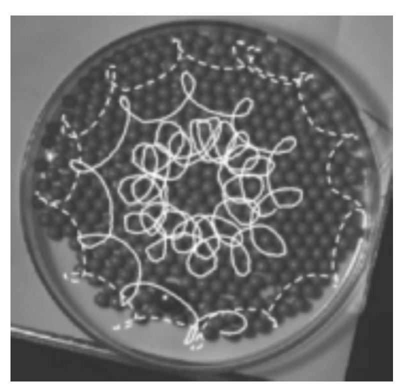

FIG. 1. A typical trajectory of the background beads in a circular plate subjected to circular vibration is shown by the dotted curve. The trajectory of an intruder (solid curve) has a cyclic spiral shape.

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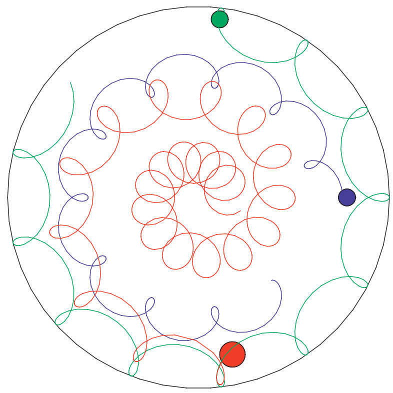

FIG. 2. (Color) Trajectory of any background bead is approximately an epitrochoid (green) when $w_p w_s < 0$ or a hypotrochoid (blue) when $w_p w_s > 0$. An intruder spirals either into the center (red) when $p < 1$ or out to the rim (not shown) when $p > 1$.

approximately a roulette, either an epitrochoid or a hypotrochoid (Fig. 2) [21]. The intruder follows a similar path except that its distance to the center of mass (CM) of the group of beads decreases with time. If we connect its average positions in each vibration period by a smooth curve, the curve would be close to a spiral. These roulette trajectories are a corollary of the fact that the whole group of beads behaves approximately as a spinning disk whose center rotates with respect to the plate center. Namely, the cluster of beads approximately forms a compact disk of radius $\rho = \sqrt{N\eta}d$, $\eta = 2\sqrt{3}/\pi \approx 1.1$ [22]. It spins with angular velocity w_s. The CM of the cluster is at the distance $R-\rho$ away from the center of the plate and rotates with angular velocity w_p (Fig. 3). The intruder, in addition to moving as a particle among the cluster, migrates in radial direction to or away from the CM. In what follows we show how the spin velocity w_s and migration velocity are related to the vibration amplitude A, the number N of small beads in the plate, and the mass and size ratios of the intruder to the background beads.

The background beads follow roulette trajectories only when their number N is large enough. They gain energy

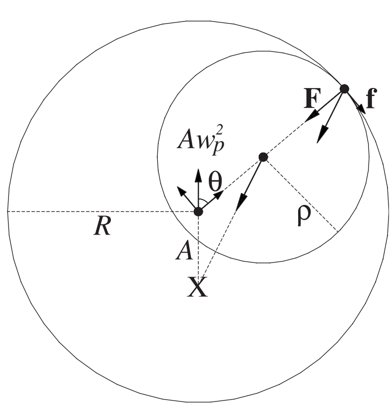

FIG. 3. The plate of radius R rotates with respect to the fixed point X with constant angular velocity w_p and amplitude A. The disk of radius ρ in the plate rotates with respect to the center of the plate with the same angular velocity w_p when it is subject to a normal force F and a tangential friction f such that both Eqs. (1) and (2) are satisfied simultaneously.

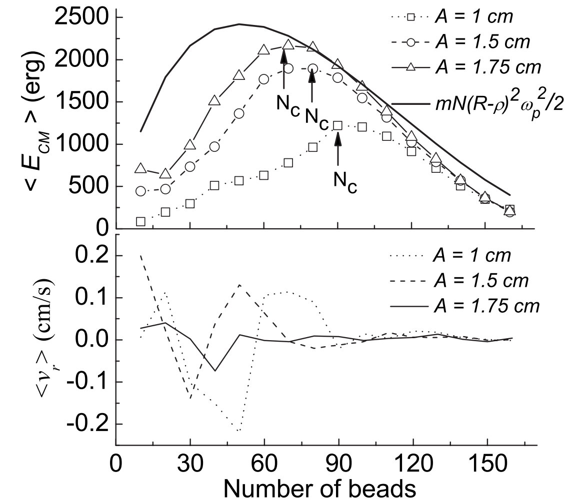

FIG. 4. When N is larger than the critical number N_c, the dissipation of energy is so large that the average radial velocity $\langle v_r \rangle$ of all beads is close to zero. Beads move in a collective way and the translational energy E_{CM} of the center of mass with respect to the plate coordinates will be given by $\frac{1}{2}Nm(R-\rho)^2 w_p^2$. The value of N_c is smaller when A is larger. (d=0.3 cm, R=4.5 cm, w_p =7.54 s^{-1}.)

when hit by the plate border and lose part of this through collisions with other beads. The energy loss due to collisions rises rapidly with the number of beads so that at some critical number the beads begin to move collectively [23]. When this happens, we observe that the whole group of beads is constantly hit by the border of the plate but its CM gains no radial velocity. This behavior can be understood by assuming that the effective restitution coefficient ε_{eff} between the group of beads and the plate border is zero when N is larger than some critical value N_c. In this case we can approximately regard the whole group of beads as clustering compactly into a single disk. In Fig. 4 we plot the average radial velocity $\langle v_r \rangle$ of all the particles and the translational energy E_{CM} of their CM with respect to the plate coordinates as a function of N. We see that when N is larger than the critical value N_c, $\langle v_r \rangle$ goes to zero and E_{CM} decreases monotonically as $\frac{1}{2}Nm(R-\rho)^2 w_p^2$, as one would expect for a disk of mass Nm rotating with radius $R-\rho$ and angular speed w_p. Increasing w_p will increase the energy at equilibrium but will not change N_c. On the other hand, increasing A will increase the frequency of collisions among beads so that ε_{eff} decreases for a fixed N. As a result, N_c is smaller when A is larger (Fig. 4).

III. ANGULAR VELOCITY OF THE WHOLE GROUP OF THE BACKGROUND BEADS

When equilibrium is reached, both the plate and the disk rotate with angular velocity w_p with respect to the center of circular vibration, with the disk lagging by a phase θ. In the meantime, the CM of the disk rotates with respect to the center of the plate with the same constant angular velocity w_p. Notice that there is a force **F** normal to the center of the plate exerted on the disk by the plate. A friction force **f**

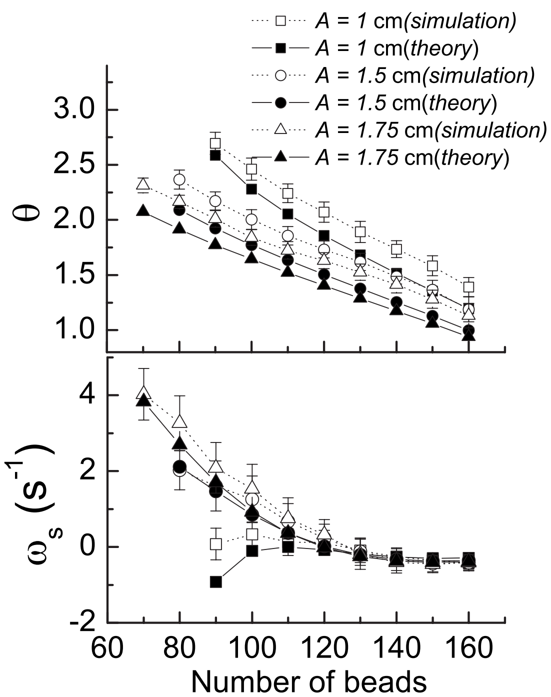

FIG. 5. Phase θ and spin w_s as a function of N for three different amplitudes. Phase and spin determined by Eq. (1) and Eq. (3), respectively, are plotted as solid curves. (d=0.3 cm, R=4.5 cm, w_p =7.54 s^{-1}.)

tangential to the rim of the disk must be present so that $\mathbf{F}+\mathbf{f}$ yields the correct centripetal force toward the center of circular vibration. On the other hand, in the plate coordinates, $\mathbf{F}+\mathbf{f}$ plus the centrifugal force of the plate toward the vibration center should also provide the correct force on the disk so that it rotates with w_p with respect to the plate center (Fig. 3). Therefore, we have

$$\frac{F}{M_d} = Aw_p^2 \cos\theta + (R-\rho)w_p^2, \tag{1}$$

$$\frac{f}{M_d} = Aw_p^2 \sin\theta, \tag{2}$$

where M_d is the mass of the disk. Assuming the friction is proportional to the relative velocity at the contact point, we have

$$\mu_{eff}[w_p(R-\rho) + w_s\rho] = Aw_p^2 \sin\theta, \tag{3}$$

where μ_{eff} is the effective friction coefficient per unit mass. Since F/M_d has limits 0 and $w_p^2 A$ at $\rho=0$ and $\rho=R$, respectively, a simple approximation for F/M_d is $F/M_d = w_p^2 A \frac{\rho}{R}$. The phase θ is then uniquely determined from Eq. (1) (θ has to be positive, otherwise the disk is not stable):

$$\cos\theta = \frac{\rho}{R} - \frac{R-\rho}{A}. \tag{4}$$

The phase calculated by Eq. (4) as a function of N is consistent with the results of our simulation (Fig. 5). Once the phase is found, the spin velocity w_s can be determined by Eq. (3). We found that our simulation results for w_s for $N>N_c$, which are qualitatively the same with the results of Ref. [18], can be obtained analytically from Eq. (3) if μ_{eff} is given by

$$\mu_{eff} = c_1 e^{c_2 N} w_p A/R, \tag{5}$$

with c_1=0.228 and c_2=0.024 (Fig. 5). Certainly we would expect that the effective friction μ_{eff} is an increasing function of N since there will be more collisions between the plate border and the beads in a vibration period when N is larger. However, the particular exponential dependence of μ_{eff} on N [Eq. (5)] needs further explanation in the future.

It is interesting to note that, for systems with vibration in one dimension, e.g., rectangular plate vibrating in longitudinal direction [24] or standard BNE under vertical vibration [1–16] friction with the container wall will normally induce convection. That is why convection was considered to be a possible cause of BNE [8]. In the circular vibration we discussed here, friction with the wall makes the whole cluster of particles rotate with respect to its center but contributes nothing to the migration of the intruder toward or away from the CM.

IV. MIGRATION DIRECTION OF THE INTRUDER

In the previous section, we were able to determine the spin of the whole group of beads by approximating them as a compact disk when N is larger than N_c. On the other hand, unless they fill the entire plate, the beads do not group together exactly like a disk. At the far end away from the contact point between the plate and the whole group of beads, the average distance between two neighboring beads could be substantially larger than $2d$. In this less dense region an intruder has the chance to change its relative radial position in the group so that migration may occur. For example, an intruder is observed to migrate toward the border if there are less and less beads between it and the border at the end of each vibration period. Note that it is crucial that, in each period, beads can most likely change their relative positions when they are all heading toward the less dense region near the border of the plate. It is due to this fact that an intruder will acquire a definite migration direction. When the beads are all moving toward the border of the plate, a large intruder, having a larger probability than its neighbors to be hit from behind, tends to migrate toward the border. On the other hand, a massive intruder, when hit, gains less velocity relative to its neighbors, therefore tends to migrate inward. In our previous work, we used a rectangular system to demonstrate that the size and the mass of the intruder play equal but opposite roles in determining the direction of the migration [24]. We argued that the parameter p, given by $p=(1+D/d)/(1+M/m)$, stands for the relative probability of the intruder to its neighbors of being hit from behind and obtaining a higher gain in speed. Therefore, the intruder will migrate toward the CM of the beads if the parameter p is $p<1$, but away from the CM if $p>1$. Our results of MD simulations for the rectangular system [24] and the circular system [25] were both consistent with the predictions of the argument. In Fig. 6 we present some experimental data for the circular system in supporting our theoretical predictions.

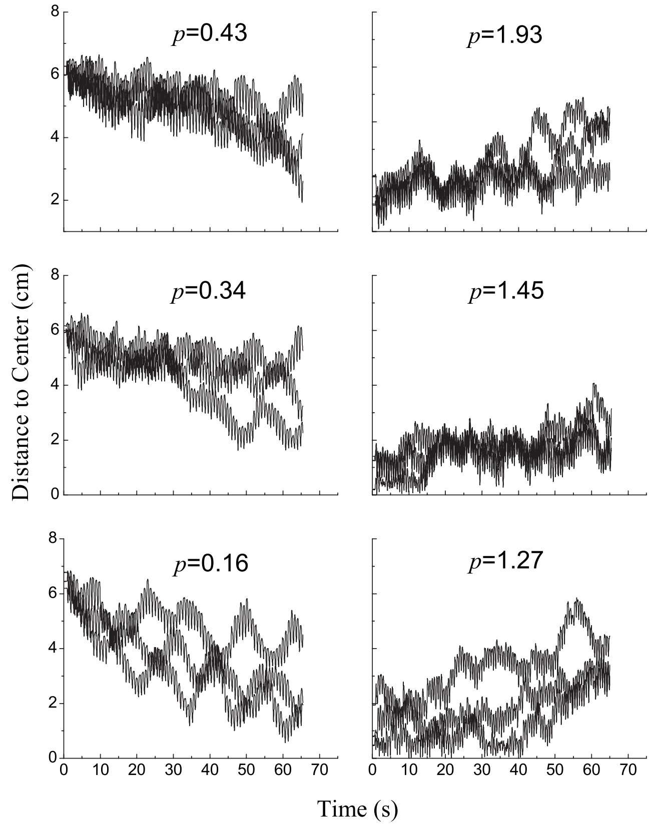

FIG. 6. Experimental data shows the intruder migrates inward for $p=0.16$, 0.34, 0.43 (left), and outward for $p=1.27$, 1.45, 1.93 (right). ($A=1.8$ cm, $R=6.9$ cm, $w_p=7.54\ \mathrm{s}^{-1}$. Background beads cover about 70% of the plate area.)

We plotted the distance of the intruder to the CM of the cluster as a function of time for several different combinations of D/d and M/m. We see that an intruder with $p>1$ indeed migrates outward, while an intruder with $p<1$ migrates inward.

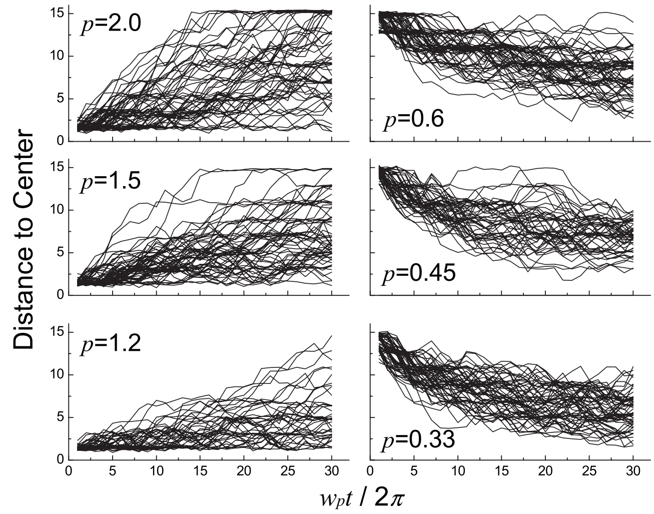

FIG. 7. Radial trajectories of the intruder using MD simulations with 60 different initial distributions for the background beads. ($N=185$, $R=17d$, $A=1$ cm, $T=1$ s, $\varepsilon=0.96$.) (Distance is in units of d.)

V. MIGRATION SPEED OF THE INTRUDER

Note the actual trajectory of an intruder varies with different initial distributions of the background beads even its tendency is predictable. In the left column of Fig. 7 we plotted the radial trajectories of the intruder in our MD simulation using 60 different initial distributions for the background beads. The initial position of the intruder is set to be near the center of the plate for $p>1$ and at border for $p<1$. The stochastic behavior of the intruder is clearly seen in Fig. 7. The average results of these 60 runs are plotted in Fig. 8. We see that the average migration path is apparently linear in time for the outward migration ($p>1$), which means the average outward-migration speed is a constant depending on the value of p only. For the inward migration, we found that the average paths are best fitted by the curves of the form $r(t)=r_0/\sqrt{1-\beta t}$, where r_0 is the initial position and β is a constant depending on p. Thus the migration speed varies as the intruder moves inward. We believe that the average spacing between beads in the neighborhood of the intruder plays an important role in determining the migration speed. A further analysis on the migration speed is currently under investigation.

VI. CONCLUSION

To conclude, we have presented a comprehensive description for the behavior of beads in the horizontal Brazil nut effect. A cluster of beads subject to circular vibration will move collectively and synchronized with the external vibration when, due to their large number or a large vibration amplitude, the collision frequency is so large that the effec-

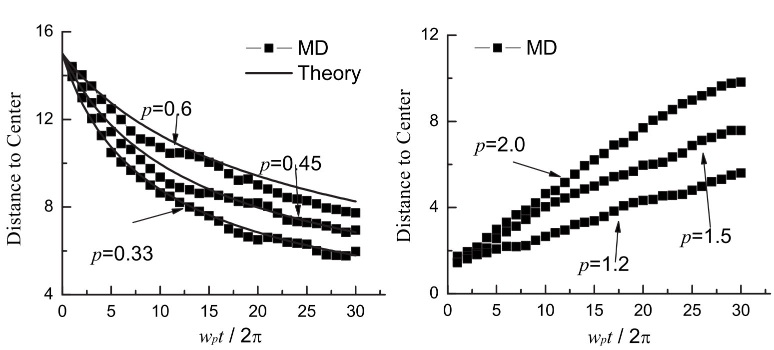

FIG. 8. Average trajectories of MD (filled square) for $p<1$ (left) and $p>1$ (right). The solid curves that fit the inward trajectories have the form $r(t)=r_0/\sqrt{1-\beta t}$. (Distance is in units of d.)

tive restitution coefficient of inelastic collisions between the plate border and the whole group of beads approaches zero. The friction force between the beads and the plate border is responsible for the rotation of the cluster with respect to its center of mass. The average trajectory of any bead in this collective motion can be approximately described by a simple geometric curve. When the vibration amplitude is not too large to prevent the beads from changing their relative positions, an intruder will move with the cluster and at the same time tend to migrate to either the center or the rim of the cluster. The migration direction depends on the mass and size ratios of the intruder to the surrounding beads and can be well predicted. The average migration speed is a constant for the outward migration, but for the inward migration, it decreases as the intruder moves toward the center of the cluster.

ACKNOWLEDGMENTS

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[22] The coverage area of a compact cluster of N beads of radius d is given by $2\sqrt{3}d^2N$, which is N times the area of a hexagon that circumscribes a circle of radius d. We approximate the cluster by an effective disk of radius ρ such that $\pi\rho^2 = 2\sqrt{3}d^2N$.

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